Math 226 Test 2

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1. (4p) Use a direct proof to show that the product of 2 odd integers is odd.
2. (6p) Prove that “If n2 + 4 is odd, then n is odd.“
3. By contraposition.
4. By contradiction.
5. (4p) Show that if . What method of proof did you use?
6. (2p) Show that for all integer values of n is false. What proof method did you use?
7. (6p) Use proof by cases to show that  is even for all integers n.
8. (4p) Determine the following values. Show all your work.
9. 
10. 
11. (4p) Give 2 positive integers and 2 negative that are congruent to -10 mod 7. Show work.
12. (5p) Show that if a,b,c and d are positive integers such that a|c and b|d then ab|cd.
13. (3p) Find the gcd(124,323) using the Euclidean algorithm. Show your work.
14. (6p) Prove by induction that:  for 
15. (3p) Write 23 in binary. Show your work.
16. (3p) Write 51751 as a hexadecimal. Show all your work.
17. (3p) Give the decimal form of the hexadecimal number .
18. (4p) Check to see if 2023 is a prime number. Explain clearly your rationale.
19. (6p) A password is required to be 6 characters long. Each character can be any one of:  
     \* 26 lowercase letters a – z  
     \* 26 uppercase letters A – Z  
     \* single digits 0 – 9

For each part, you may leave your answer as an expression.

1. With no restrictions, how many 6-character passwords are possible?
2. How many 6-character password are possible if the first letter must be a capital letter and no other letters can be capital and the last 2 characters must be numbers. The second through 4th must be lower case letters and repetition is not permitted.
3. (3p) How many bit strings of length 7 contain exactly 3 ones?
4. (3p) In how many ways can a set of 4 letters be selected from the set of letters in the English alphabet?
5. (3p) What is the coefficient of in the expansion of ?
6. (3p) How many possible solutions are there to the equation if only non-zero integer solutions are allowed?
7. (3p) How many combinations of cookies can you choose if you are buying 5 cookies, there are 7 kinds available and the first of the 5 cookies you pick must be a chocolate chip cookie. The cookies don’t have to be different (repeats are allowed).
8. (6p) You roll 2 six-sided dice find the following probabilities:
9. Rolling at least a four?
10. Rolling no fours?
11. Rolling a sum of 5?
12. (6p) Of 200 people surveyed:

120 people like Coca-Cola  
 90 people like Pepsi  
 40 people like both.

a)What is the probability of selecting someone who likes Coca-Cola or Pepsi?

b) What is the probability of selecting someone who likes neither?

1. (7p) Assume the probability of a family having a boy is twice as likely as the probability of the family having a girl.
2. What is the probability of having a boy?
3. What is the probability of having a girl?
4. If the family will have 2 children, give all the possible outcomes. Let B be a boy and G be a girl.
5. What is the probability of the family having two boys given that the first one was a boy?
6. (3p) When flipping two coins, are the probabilities of flipping one head and one tail independent? Explain.

1. Extra credit (2p) : Find the inverse of 5 modulo 3. Show your work.